

# OPEN-LOOP CONTROL IN QUANTUM OPTICS: TWO-LEVEL ATOM IN MODULATED OPTICAL FIELD

Saifullah<sup>1</sup>, Sergei Borisenok<sup>2</sup>

<sup>1</sup>Abdus Salam School of Mathematical Sciences,  
Government College University,  
35 C - II, Gulberg III, Lahore, Pakistan

Email: saifullahkhalid75@yahoo.com

<sup>2</sup> Deptt. of Physics, Herzen State Pedagogical University  
48 Moika River, 191186 St. Petersburg, Russia

Email: sebori@mail.ru

**ABSTRACT.** The methods of mathematical control theory are widely used in the modern physics, but still they are less popular in quantum science. We will discuss the aspects of control theory, which are the most useful in applications to the real problems of quantum optics. We apply this technique to control the behavior of the two-level quantum particles (atoms) in the modulated external optical field in the frame of the so called "semi classical model", where quantum two-level atomic system (all other levels are neglected) interacts with classical electromagnetic field. In this paper we propose a simple model of feedforward (open loop) control for the quantum particle system, which is a basement for further investigation of two-level quantum particle in the external one-dimensional optical field.

## 1. INTRODUCTION

In this paper we present a semiclassical theory of the interaction of a single two-level atom with a single mode of the field in which the atom is treated as a quantum two-level system and the field is treated classically. We will use a simple physical model in order to define the goals, terminology, and methodology of applied control theory.

We propose a simple model of feedforward (open-loop) control for two-level atom in the external one-dimensional optical field.

We use the so-called "semi-classical model" of the atom-field interaction

that describes a single quantum two-level atomic system (all other levels are neglected) with classical electromagnetic field. We use the standard notation following [1], but in our model the optical field plays the role of a control signal  $u(t)$  for open-loop (feedforward) control scheme [2]. Recently other authors studied the control of two-level atoms in the frame of open-loop ideology when the controlling field was known a priori. It allowed to get the different forms of atomic energy spectra, producing  $\pi$ - and  $\frac{\pi}{2}$ - pulses [3], taking special non-constant shapes of external field [4] etc.

## 2. INTERACTION OF TWO-LEVEL ATOM WITH OPEN LOOP CONTROL OPTICAL FIELD

We consider the quantum two-level atomic system in the classical optical field  $E(t)$ . Let  $|a\rangle$  and  $|b\rangle$  represent the upper and lower level states of the atom, i.e., they are eigenstates of the unperturbed part of the Hamiltonian  $\hat{H}_0$  with the eigenvalues:  $\hat{H}_0|a\rangle = \hbar\omega_a|a\rangle$  and  $\hat{H}_0|b\rangle = \hbar\omega_b|b\rangle$ .

The equations of motion for the density matrix elements are given by [2]:

$$\begin{aligned}\dot{\rho}_{aa} &= -\gamma_a\rho_{aa} + \frac{\iota E}{\hbar} \left( P_{ab}\rho_{ba}e^{\iota\omega t} - P_{ab}^*\rho_{ab}e^{-\iota\omega t} \right); \\ \dot{\rho}_{bb} &= -\gamma_b\rho_{bb} - \frac{\iota E}{\hbar} \left( P_{ab}\rho_{ba}e^{\iota\omega t} - P_{ab}^*\rho_{ab}e^{-\iota\omega t} \right); \\ \dot{\rho}_{ab} &= -\gamma_{ab}\rho_{ab} - \frac{\iota E}{\hbar} P_{ab} \left( \rho_{aa} - \rho_{bb} \right) e^{\iota\omega t},\end{aligned}\tag{1}$$

where  $\rho_{ba} = \rho_{ab}^*$ ;  $P_{ab}$  is the matrix element of the electric dipole moment,  $\gamma_a$  and  $\gamma_b$  are the decay constants,  $\gamma_{ab} = \frac{\gamma_a + \gamma_b}{2} + \gamma_{ph}$  is a decay rate including elastic collisions between atoms, and  $\omega = \omega_a - \omega_b$  is the atomic transition frequency.

Let's denote  $P_{ab} = |P_{ab}|e^{\iota\varphi}$  and

$$\begin{aligned}\rho_+ &= \rho_{ba}e^{\iota(\omega t + \varphi)} + \rho_{ab}e^{-\iota(\omega t + \varphi)}; \\ \rho_- &= \iota \left[ \rho_{ba}e^{\iota(\omega t + \varphi)} - \rho_{ab}e^{-\iota(\omega t + \varphi)} \right].\end{aligned}\tag{2}$$

Using (2) we can re-write the system (1) in the real form:

$$\begin{aligned}\dot{\rho}_{aa} &= -\gamma_a \rho_{aa} + \frac{|P_{ab}|E}{\hbar} \rho_- ; \\ \dot{\rho}_{bb} &= -\gamma_b \rho_{bb} - \frac{|P_{ab}|E}{\hbar} \rho_- ; \\ \dot{\rho}_+ &= -\gamma_{ab} \rho_+ + \omega \rho_- ; \\ \dot{\rho}_- &= -\gamma_{ab} \rho_- - \omega \rho_+ - \frac{2|P_{ab}|E}{\hbar} (\rho_{aa} - \rho_{bb}).\end{aligned}\tag{3}$$

For further calculation we put  $\gamma_a = \gamma_b = \gamma$ .

Then

$$(\rho_{aa} + \rho_{bb})(t) = e^{-\gamma t} (\rho_{aa} + \rho_{bb})(0).\tag{4}$$

The first two equations of system (3) can be combined together.

We put:

$$\begin{aligned}\rho_{aa} - \rho_{bb} &\equiv e^{-\gamma t} x(t) ; \\ \rho_+ &\equiv e^{-\gamma t} y(t) ; \\ \rho_- &\equiv e^{-\gamma t} z(t).\end{aligned}\tag{5}$$

Substituting (5) in (3) we eliminate the decay  $\gamma$ -containing terms.

Finally, re-scaling the time by  $\omega$ :  $\tau = \omega t$ , and denoting the dimensionless control signal by  $u(t) \equiv \frac{2|P_{ab}|E(t)}{\hbar\omega}$  and  $\epsilon = \frac{\gamma\hbar}{\omega}$ , we get the simplified system:

$$\begin{aligned}\dot{x} &= u(t)z ; \\ \dot{y} &= -\epsilon y + z ; \\ \dot{z} &= -\epsilon z - y - u(t)x.\end{aligned}\tag{6}$$

Here the dot means the derivative with respect to the new dimensionless time  $\tau$ .

We remind that  $x \in [-1, 1]$ , since  $\rho_{aa} - \rho_{bb} \in [-1, 1]$ , and  $\rho_{aa} - \rho_{bb} \rightarrow 0$  as  $t \rightarrow \infty$ .

### 3. THE CASE OF CONSERVATIVE SYSTEM

We mention here the special (particular) case of  $\gamma_{ph} = 0$  and thus,  $\epsilon = 0$ . Then the system (6) has the integral of motion:

$$x^2 + y^2 + z^2 \equiv r^2 = \text{const}.\tag{7}$$

Now in the spherical coordinates:

$$\begin{aligned} x(\tau) &= r \cos \alpha(\tau) \sin \beta(\tau) ; \\ y(\tau) &= r \sin \alpha(\tau) \sin \beta(\tau) ; \\ z(\tau) &= r \cos \beta(\tau). \end{aligned} \tag{8}$$

Then

$$\begin{aligned} \dot{\alpha} &= \cot \beta \left( \cos \alpha - u \sin \alpha \right); \\ \dot{\beta} &= \sin \alpha + u \cos \alpha. \end{aligned} \tag{9}$$

If  $x(0) = -1$ ,  $y(0) = z(0) = 0$ , then  $\alpha(0) = 0$  and  $\beta(0) = \frac{-\pi}{2}$ .

#### 4. NUMERICAL SOLUTION

To solve (6) numerically we will use Maple.

Let's apply the initial conditions

$$x(0) = -1, \quad y(0) = z(0) = 0,$$

corresponding to the ground level of the atom.

Here we put  $\epsilon = 0.01$  and  $u(\tau) = \cos \tau$ .

The result is presented on Fig.1 (left).

Another case corresponds to the increasing signal  $u(\tau) = e^{-\tau} \cos \tau$ .

The result is presented on Fig.1 (right).

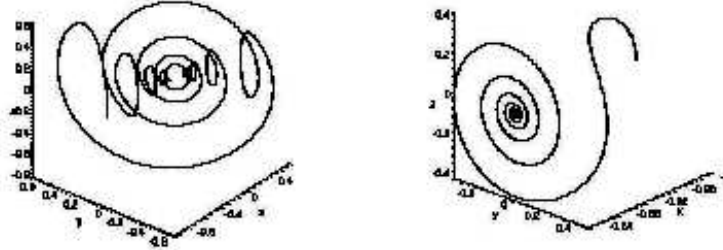


FIGURE 1. Phase portraits of the dynamical system (6) with  $\epsilon = 0.01$  and the control signal  $u(\tau) = \cos \tau$  (left) and  $u(\tau) = e^{-\tau} \cos \tau$  (right).

From Fig.1 we can conclude that the shape of the control field (i.e. the

optical field  $E$ ) influences drastically on the present state of two-level atoms.

## 5. CONCLUSION

In this paper we studied the techniques to control the behavior of the two-level quantum particles by the modulated external optical field.

We conclude that the semi-classical model of interaction between two-level quantum particle system and classical optical field can be successfully applied to describe the control process of the particle energy stabilization.

## REFERENCES

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